$\frac{39.5}{50}$

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20911132 10 – 11

Question 1: (10 points)

Circle the correct answer:

1) If $p(x) = (x+1)^4 (x-1)^3$ then 1 is a zero of multiplicity:

a) 3 b) 4 c) 7 d) 1

2) The sequence $\left\{2^{-3^n}\right\}_{n=1}^{\infty}$ converges to :

a) zero linearly b) zero cubically c) zero quadratically d) one linearly

3) Let $p_0=2$, $p_1 = 2.1$, $p_2 = 2.2$ then $\Delta^2 p_0 =$:

$2.2 - 2(2.1) + 2$

a) $0.\bar{1}$ b) 0.4 c) 0.2 d) 0

4) Let $x_0=2$, $x_1 = 2.5$, then the first interpolating polynomial of $f(x)= x^2$ is:

a) -30.5-27 b) 30.5x+27 c) 4.5x-5 d) -4.5x+5

7.5

$\frac{2^{-3^{n+1}}}{2^{-3^n \cdot 3}}$

$\frac{x-2.5}{2-2.5}(4) + \frac{(x-2)}{(2.5-2)} 6.25$

$\frac{x-2.5}{-0.5} 4 + \frac{(x-2)}{0.5} 6.25$

$\frac{4x-10}{-0.5} + \frac{6.25x-12.5}{0.5}$

$-8x+20+12.5x-25$

$4.5x-5$

$-8x+20+12.5x-25 = 4.5x-5$

Question 2 : (20 points)

a) Find an approximation " x_2 " to one of the zero's of $p(x) = x^3 - x - 1$ using Newton-Raphson and synthetic division where $x_0 = 1$ (using four digit rounding) .

$\boxed{P(x) = x^3 - x - 1}$

	x^3	x^2	x^1	x^0
	1	0	-1	-1
1		1	1	0
	1	1	0	-1

$\to f(1) = -1$

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$\boxed{\therefore f(1) = -1}$

	x^2	x^1	x^0
	1	1	0
1		1	2
	1	2	2

$\to f'(1) = 2$

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$\Rightarrow \boxed{f'(1) = 2}$

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by Newton Raphson Method in

$$P_1 = P_0 - \frac{f(P_0)}{f'(P_0)}$$

$$\therefore P_1 = 1 - \frac{f(1)}{f'(1)} = 1 - \left(\frac{-1}{2}\right) = 1 - -\frac{1}{2} = 1 + \frac{1}{2} = 3/2 = \boxed{1.5}$$

	x^3	x^2	x^1	x^0
	1	0	-1	-1
1.5		1.5	2.25	1.875
	1	1.5	1.25	0.875

$0.875 \Rightarrow f(1.5) = \boxed{0.875}$

$\therefore \boxed{f(1.5) = 0.875}$

	x^2	x^1	x^0
	1	1.5	1.25
1.5		1.5	4.5
	1	3	5.75

$\Rightarrow f'(1.5) = \boxed{5.75}$

$\therefore f'(1.5) = 5.75$

$$\therefore X_2 = X_1 - \frac{f(X_1)}{f'(X_1)} = 1.5 - \frac{f(1.5)}{f'(1.5)}$$

$$= 1.5 - \frac{0.875}{5.75}$$

$$= \boxed{1.3478}$$

b) Suppose that the Taylor series for a function is

$$f(x) = x + \frac{1}{2}x^3 + \frac{1}{3}x^5 + \frac{1}{4}x^7 + ..$$

And $g(x) = 1 - x^2 + x^4 - x^6 + ...$

, consider $F(x) = f(x) - xg(x)$, find the multiplicity of the root x=0 .

$f(x) = x^1 + \frac{1}{2}x^3 + \frac{1}{3}x^5 + \frac{1}{4}x^7$

$g(x) = 1 - x^2 + x^4 - x^6 + --$

$x\,g(x) = x - x^3 + x^5 - x^7 + ---$

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$f(x) - x\,g(x) = (\cancel{x} + \frac{1}{2}x^3 + \frac{1}{3}x^5 + \frac{1}{4}x^7) - (\cancel{x} - x^3 + x^5 - x^7) ---$

$= -\frac{1}{2}x^3 + \frac{4}{3}x^5 - \frac{3}{4}x^7 + ---$

$\Rightarrow$ Taylor series : $1 + f'(x)(x-x_0)^1 + \frac{f''(x)(x-x_0)^2}{2!} + \frac{f'''(x)(x-x_0)^3}{3!}$

$\frac{1}{2}\cancel{x}^3 = \frac{f'''(x)\cancel{x}^3}{6}$ $\quad \frac{f'''(x)}{6} = \frac{1}{2}$ $\quad f'''(x) = \frac{6}{2} = \boxed{3}$

$f''(x) = 3x + c \Rightarrow f'(x) = \frac{3x^2}{2} + c \Rightarrow f(x) = \frac{3x^3}{2 \cdot 3} = \frac{3x^3}{6}$

$f(x) = \boxed{\frac{x^3}{2}}$

$g(x)$:- $-\cancel{x}^2 = \frac{f''(x)\cancel{x}^2}{2} \Rightarrow -1 = \frac{f''(x)}{2}$

$f''(x) = -2 \Rightarrow f'(x) = -2x \Rightarrow f(x) = -\frac{2x^2}{2} = \boxed{-x^2}$

$x\,g(x) = x(-x^2) = -x^3$

$\therefore F(x) = f(x) - x\,g(x) = \frac{x^3}{2} - -x^3 = \frac{x^3}{2} + x^3$

$= \boxed{\frac{3x^3}{2}}$

$\therefore F(x)$ has a multiplicity of $\boxed{3}$ at $x = 0$

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Question 3: (20 points)

The following table lists the values of f(x) = $\sqrt{x}$ accurate to the places given ,
$x \in [1.2, 1.6]$

	X_i	$F(X_i)$
x_0	1.2	1.095
x_1	1.4	1.183
x_2	1.6	1.265

a) Use Nevill's method to approximate f(1.5).
b) Find the actual error.
c) Find the upper bound of the error.

a)

$Q_{00} = P_0 = f(x_0)$

$Q_{10} = P_1 = f(x_1)$

$Q_{20} = P_2 = f(x_2)$

$$Q_{11} = P_{01} = \frac{(x - x_0)P_1 - (x - x_1)P_0}{x_1 - x_0}$$

$$Q_{11}(1.5) = \frac{(x-1.2)(1.183) - (x-1.4)(1.095)}{1.4-1.2}$$

$$= \frac{(1.5-1.2)(1.183) - (1.5-1.4)(1.095)}{0.2}$$

$$= \frac{(0.3)(1.183) - (0.1)(1.095)}{0.2} = \boxed{1.227}$$

$$Q_{21} = P_{12} = \frac{(x - x_1)P_2 - (x - x_2)P_1}{x_2 - x_1}$$

$$Q_{21}(1.5) = \frac{(1.5-1.4)(1.265) - (1.5-1.6)(1.183)}{1.6-1.4}$$

~~Good Luck~~

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$$= \frac{(0.1)(1.265) - (-0.1)(1.183)}{0.2} = \boxed{1.224}$$

$$Q_{22} = P_{012} = \frac{(x - x_0) P_{12} - (x - x_2) P_{01}}{x_2 - x_0}$$

$$= \frac{(1.5 - 1.2)(1.224) - (1.5 - 1.6)(1.227)}{1.6 - 1.2}$$

$$= \frac{(0.3)(1.224) + (0.1)(1.227)}{0.4} = \boxed{1.22475}$$

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(b) Actuall error $= |f(1.5) - Q_{22}(1.5)|$

$f(1.5) = \sqrt{1.5} = \boxed{1.22474}$

$Q_{22}(1.5) = \boxed{1.22475}$

$\Rightarrow$ Actuall error $= |1.22474 - 1.22475| = \boxed{0.00001} = \boxed{1 \times 10^{-5}}$

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(c) upper bound

$$= \left| \frac{f'''(\xi)}{3!} (x - x_0)(x - x_1)(x - x_2) \right|$$

$f(x) = \sqrt{x} = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$

$f''(x) = \frac{-1}{4} x^{-\frac{3}{2}}$

$f'''(x) = \frac{+3}{8} x^{-\frac{5}{2}} \Rightarrow \therefore f'''(x) = \frac{3}{8} x^{-\frac{5}{2}} = \frac{3}{8} \frac{1}{\sqrt{x^5}} = \frac{3}{8\sqrt{x^5}}$

$$\therefore \left| \frac{f'''(\xi)}{6} (x - x_0)(x - x_1)(x - x_2) \right| = \left(\frac{\cancel{3}}{\cancel{6}_2 \cdot 8\sqrt{x^5}} (x - 1.2)(x - 1.4)(x - 1.6) \right|$$

$$= \left| \frac{1}{16\sqrt{\xi^5}} (x - 1.2)(x - 1.4)(x - 1.6) \right| = \left| \frac{1}{16\sqrt{\xi^5}} (1.5 - 1.2)(1.5 - 1.4)(1.5 - 1.6) \right|$$

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$$= \left| \frac{1}{16\sqrt{\xi^5}} (0.3)(0.1)(-0.1) \right| = \left| \frac{-0.003}{16\sqrt{\xi^5}} \right|$$

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take $\xi = 1.2$ ($\boxed{\text{the smaller value}}$) $\Rightarrow \left| \frac{-0.003}{16\sqrt{\xi^5}} \right| \leq \left| \frac{0.003}{16\sqrt{(1.2)^5}} \right|$

$$= \left| \frac{-0.003}{16(1.577)} \right| = \left| \frac{-0.003}{25.232} \right| = |-1.188 \times 10^{-4}| = \boxed{1.188 \times 10^{-4}}$$